Analysing and Understanding Learning Assessment for Evidence-based Policy Making

Introduction to Hierarchical Linear Model
Bangkok, 14-18, Sept. 2015
Single-level regression

- Linear regression equation consists of intercept, regression coefficients, explanatory factors and error variance:

\[ Y_i = \alpha + \beta_1 X_1 + \beta_2 X_2 + e_i \]

- However, effects could be different at different levels
Single-level regression

\[ y = 0.4479x + 46.218 \]
Multi-level regression
Multi-level regression

"Grand" Intercept with group variation & "Grand" Slope with group variation

 Variation in intercepts around the "grand" intercept

 Variation in slopes around the "grand" slope

 "Grand" Intercept

 "Grand" Slope

 Cluster 1

 Cluster 2

 Cluster 3
Basic multi-level notation

- With student (level 1) nested within schools (level 2) one could assume a hierarchical model
  - Level-1 model:
    \[ Y_{ij} = \beta_{0j} + \beta_{1j}X_{ij} + r_{ij} \]
  - Level-2 model:
    \[ \beta_{0j} = \gamma_{00} + u_j \]
    \[ \beta_{1j} = \gamma_{10} \]
  - This can also be written as a “mixed” model:
    \[ Y_{ij} = \gamma_{00} + \gamma_{10}X_{ij} + u_j + r_{ij} \]
Linear versus ML regression
Linear versus ML regression
Linear versus ML regression
Multi-level equations:

one student background variable as fixed effect

\[ Y_{ij} = \alpha_j + \beta X_{ij} + \varepsilon_{ij} \]

\[ \alpha_j = \gamma_{00} + U_{0j} \]

\[ \beta = \gamma_{10} \]
Random slopes model

![Graph showing math performance vs. SES with four different slopes labeled Sc1 to Sc4.](image)
ML equations:
one student background variable
random effect

\[ Y_{ij} = \alpha_j + \beta_j X_{ij} + \varepsilon_{ij} \]

\[ \alpha_j = \gamma_{00} + U_{0j} \]

\[ \beta_j = \gamma_{10} + U_{1j} \]
Fixed versus Random factors

\[ Y_{ij} = \alpha_j + \beta X_{ij} + \epsilon_{ij} \]
\[ \alpha_j = \gamma_{00} + U_{0j} \]
\[ \beta = \gamma_{10} \]

\[ Y_{ij} = \alpha_j + \beta_j X_{ij} + \epsilon_{ij} \]
\[ \alpha_j = \gamma_{00} + U_{0j} \]
\[ \beta_j = \gamma_{10} + U_{1j} \]
School and within school variance for mathematics performance in PISA 2003
Example (Germany PISA 2003)

### Estimates of Covariance Parameters$^{a,b}$

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimate</th>
<th>Std. Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Residual</td>
<td>4498.698</td>
<td>95.446437</td>
</tr>
<tr>
<td>Intercept [subject variance]</td>
<td>6237.027</td>
<td>624.5138</td>
</tr>
</tbody>
</table>

a. Dependent Variable: pv1math.
b. Residual is weighted by normwgt.

\[
\rho = \frac{\sigma_{\text{school}}^2}{\sigma_{\text{school}}^2 + \sigma_{\text{within\_school}}^2} = \frac{6237}{6237 + 4499} = 0.58
\]
Intra-class correlations for mathematics performance in PISA 2003
Some observations

• Coefficients in MLM are averages of within-school estimates:
  – Intercept: Average school mean after controlling for predictors
  – Student-level coefficients: Average within-school effects (fixed or random)
  – School-level coefficients: Effect of school-level factors on school means

• Variance components at each level

• MLM assumes simple random samples at each level
Comparison of OLS vs. ML regression

<table>
<thead>
<tr>
<th>Regression Type</th>
<th>ISL</th>
<th>USA</th>
<th>DEU</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear regression coefficient</td>
<td>1.17</td>
<td>2.03</td>
<td>2.78</td>
</tr>
<tr>
<td>Fixed effect multilevel regression coefficient</td>
<td>1.04</td>
<td>1.28</td>
<td>0.86</td>
</tr>
<tr>
<td>Random effect multilevel regression coefficient</td>
<td>1.04</td>
<td>1.22</td>
<td>0.83</td>
</tr>
</tbody>
</table>
Gender regression coefficient: linear versus ML coefficient:
Pisa 2003 mathematics

![Graph showing gender regression coefficients for AUT, CHE, and NOR countries comparing ML and Linear methods.](image-url)
Time on Homework coefficient: linear versus ML coefficient:

Pisa 2000 reading
Interpretation

• Within-school coefficients have a different interpretation from those obtained from a single-level regression!

• In countries with homogeneous grouping of students, low within-school coefficients are to be expected

• With heterogeneous grouping of students, low between-school coefficients are to be expected
When to Use MLM

- MLM is appropriate only if you have a *conceptual model* that requires modelling of different levels
- Single-level regression provides highly similar results as MLM with only fixed coefficients
  - However, no estimation of variance components at each level
- Often *not useful* when your variables have hardly any between-cluster variance